3.1 instantaneous and average rates of change

2014-10-01 day 27

7) $y=\frac{1}{2}x^2$; $x_0=3, x_1=4$

a) avg rate of chg $= \frac{y(4)-y(3)}{4-3} = \frac{8-\frac{9}{2}}{1} = \frac{7}{2}$

b) instantaneous r.o.c. at $x_0=3$ $= \lim_{x\to 3} \frac{y(x)-y(3)}{x-3} = \lim_{x\to 3} \frac{\frac{1}{2}x^2-\frac{9}{2}}{x-3}$

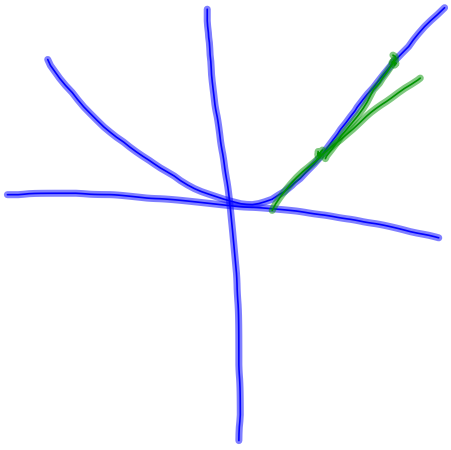
$= \frac{1}{2}\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{1}{2}\lim_{x\to 3} x+3 = \frac{1}{2}(6) = 3$

c) instantaneous r.o.c. at x_0 (generally) $= \lim_{x\to x_0} \frac{y(x)-y(x_0)}{x-x_0} = \lim_{x\to x_0} \frac{\frac{1}{2}x^2-\frac{1}{2}x_0^2}{x-x_0}$

$= \frac{1}{2}\lim_{x\to x_0} \frac{x^2-x_0^2}{x-x_0} = \frac{1}{2}\lim_{x\to x_0} \frac{(x-x_0)(x+x_0)}{x-x_0}$

$= \frac{1}{2}\lim_{x\to x_0} x+x_0 = \frac{1}{2}(x_0+x_0) = x_0$

IE. the slope of the tangent line



3.1 instantaneous and average rates of change

2014-10-01 day 27

a) $y=\frac{1}{x}$; $x_0=2$, $x_1=3$

$$\text{avg} = \frac{\frac{1}{3}-\frac{1}{2}}{3-2} = \frac{\frac{2}{6}-\frac{3}{6}}{1} = \frac{-\frac{1}{6}}{1} = -\frac{1}{6}$$

b) inst. @ $x_0=2$

$$= \lim_{x\to 2} \frac{y(x)-y(2)}{x-2} = \lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$$

$$= \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} \frac{-(x-2)}{2x(x-2)}$$

$$= \lim_{x\to 2} -\frac{1}{2x} = -\frac{1}{4}$$

$$\frac{1}{x}-\frac{1}{x_0} = \frac{x_0}{xx_0}-\frac{x}{xx_0}$$

c) inst @ x_0

$$= \lim_{x\to x_0} \frac{y(x)-y(x_0)}{x-x_0} = \lim_{x\to x_0} \frac{\frac{1}{x}-\frac{1}{x_0}}{x-x_0}$$

$$= \lim_{x\to x_0} \frac{\frac{x_0-x}{xx_0}}{x-x_0} = \lim_{x\to x_0} \frac{-(x-x_0)}{xx_0(x-x_0)}$$

$$= \lim_{x\to x_0} \frac{-1}{xx_0} = \frac{-1}{x_0x_0} = -\frac{1}{x_0^2}$$

$x_0-x = -(x-x_0)$

useful tool

$$\frac{\frac{a}{b}\left(\frac{1}{c}\right)}{\left(\frac{c}{1}\right)\left(\frac{1}{c}\right)}$$

3.1 instantaneous and average rates of change

2014-10-01 day 27

10. b $\lim_{x\to 1} \dfrac{\frac{1}{x^2}-\frac{1}{1^2}}{x-1} = \lim_{x\to 1} \dfrac{\frac{1}{x^2}-\frac{x^2}{x^2}}{x-1} =$

$1/x^2$, $x_0 \neq 1, x_1 = 2$

$= \lim_{x\to 1} \dfrac{\frac{1-x^2}{x^2}\times\frac{1}{x-1}}{x-1} = \lim_{x\to 1} \dfrac{1-x^2}{x^2(x-1)} =$

$= \lim_{x\to 1} \dfrac{(1-x)(1+x)}{x^2(x-1)} = \lim_{x\to 1} \dfrac{-(-1+x)(1+x)}{x^2(x-1)} = \lim_{x\to 1} \dfrac{-(1+x)}{x^2} = \dfrac{-1-1}{1^2} = \dfrac{-2}{1} = -2$

c) $\lim_{x\to x_0} \dfrac{\frac{1}{x^2}-\frac{1}{x_0^2}}{x-x_0} = \lim_{x\to x_0} \dfrac{\frac{x_0^2-x^2}{x^2x_0^2}}{x-x_0} = \lim_{x\to x_0} \dfrac{x_0^2-x^2}{(x-x_0)(x^2x_0^2)}$ ($y(x)$, $y(x_0)$)

E $= \lim_{x\to x_0} \dfrac{(x_0-x)(x_0+x)}{(x-x_0)(x^2x_0^2)} = \lim_{x\to x_0} \dfrac{-(x_0+x)}{x^2x_0^2}$

$= \dfrac{-(x_0+x_0)}{x_0^2x_0^2} = \dfrac{-2x_0}{x_0^4} = \boxed{\dfrac{-2}{x_0^3}}$

3.1 instantaneous and average rates of change

2014-10-01 day 27

$$x^3 - y^3 = (x-y)(x^2+xy+y^2)$$

$$\begin{array}{r} x^2+x+1 \\ x-1 \\ \hline -x^2-x-1 \\ x^3+x^2+x \\ \hline x^3 \qquad -1 \end{array}$$